# TRUCK PERFORMANCE CURVES REFLECTIVE OF TRUCK AND PAVEMENT CHARACTERISTICS

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## ABSTRACT

The paper utilizes a linearly increasing variable power vehicle dynamics model to develop the TRUCKSIM software for the modeling of truck acceleration behavior along grade sections. The TRUCKSIM software is demonstrated to produce truck performance curves that are consistent with the HCM procedures for a 120 kg/kW (200 lb/hp) truck equipped with radial tires traveling on a fair asphalt surface (Pavement Serviceability Index between 1.5 and 3.0). Using the software, the sensitivity of truck performance curves to roadway and truck characteristics are quantified. Subsequently, truck performance curves that are reflective of in-field truck characteristics are developed. These truck performance curves are intended to enhance the Highway Capacity Manual (HCM) procedures in locating truck climbing lanes.

**Key words**: Truck modeling, vehicle dynamics, roadway design, truck climbing lanes.

## INTRODUCTION

Mannering and Kilareski (1998) suggest that "*the performance of road vehicles forms the basis for highway design guidelines and traffic analysis. For example, in highway design, determination of the length of freeway acceleration and deceleration lanes, maximum highway grades, stopping-sight distances, passing-sight distances, and numerous accident prevention devices all rely on the basic understanding of vehicle performance*."

Truck performance along grade sections may have significant impacts on roadway throughput and efficiency depending on the roadway grade, the truck characteristics, the percentage of trucks on the roadway, and the overall level of congestion on the roadway section. Although the Highway Capacity Manual (HCM) and the American Association of State Highway and Transportation Officials (AASHTO) Geometric Design Guide provide curves for predicting vehicle speeds as a function of the distance traveled and the percentage grade along a roadway section (TRB, 2002 and AASHTO, 1994), these curves suffer from a number of shortcomings. First, Figure 1 illustrates that the truck performance curves do not cover speeds that exceed 90 km/h (55 mi/h). Second, the HCM procedures are limited because the curves only cover a single truck weight-to-power (W/P) ratio of 120 kg/kW (200 lb/hp). Third, the curves do not capture the effect of different pavement types, pavement surface conditions, and the truck characteristics on the tuck acceleration behavior. Forth, the execution of the curves is time consuming and could be refined by automating the process.

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## Research Objectives and Research Significance

The objectives of this paper are three-fold. First, the paper develops the TRUCKSIM software for identifying locations of truck climbing lanes based on user defined truck, tire, and roadway parameters. It is envisioned that the software would be of significant assistance to practitioners in automating the truck climbing lane design process. Second, the paper investigates the impact of pavement type, pavement condition, truck weight, and truck power characteristics on truck acceleration behavior. It is hypothesized that the truck and pavement characteristics will significantly affect truck acceleration behavior along grade sections because of changes in the truck power, rolling resistance coefficients, and the coefficient of friction. Third, the paper expands the domain of application of the Highway Capacity Manual (HCM) truck performance curves by considering different truck weight-to-power ratios, roadway pavement types, the pavement conditions.

The significance of this research effort lies in the fact that it develops the TRUCKSIM software for locating truck climbing lanes optimally along highway sections. The software, which can be run on a personal computer, is sensitive to truck, tire, and roadway surface conditions. Using the TRUCKSIM software, the research effort characterizes the impact of truck and pavement characteristics on maximum truck acceleration behavior. Subsequently, the paper develops truck performance curves that are more reflective of current in-field truck, roadway, and tire characteristics. The proposed performance curves can be utilized to enhance the Highway Capacity Manual (HCM) truck performance procedures in order to reflect advances in truck engines since the curves were initially developed. Furthermore, the research extends the HCM truck performance curves by incorporating truck and pavement characteristics that are not accounted for in the state-of-practice HCM performance curves.

## Paper Layout

Initially, the vehicle dynamics model that is incorporated in the TRUCKSIM software and utilized to develop the truck performance curves that are presented in the paper is described briefly. Subsequently, the importance of enhancing the state-of-practice truck performance curves is demonstrated through a number of simple example illustrations. The following section describes the logic of the TRUCKSIM software followed by a description of how truck performance curves for different truck and pavement characteristics are developed using the TRUCKSIM software. Specifically, the input parameters that are utilized to develop the curves are described. Subsequently, the applicability of the TRUCKSIM software is demonstrated by applying the software to a 45-km segment of I-81 in the state of Virginia. Finally the conclusions of the paper together with recommendations for further research are presented.

# VEHICLE DYNAMICS MODEL

Vehicle dynamics models compute maximum vehicle acceleration levels by computing the resultant force acting on a vehicle, as summarized in Equation 1. Given that acceleration is the second derivative of distance with respect to time, Equation 1 resolves to a second-order Ordinary Differential Equation (ODE) of the form presented in Equation 2.

The vehicle tractive effort is computed using Equation 3 with a maximum value based on Equation 4, as demonstrated in Equation 5. Equation 4 ensures that the tractive force does not exceed the maximum frictional force that can be sustained between the vehicle's tractive-axle tires and the roadway surface without the spinning of the vehicle wheels. The equation demonstrates that the maximum tractive force is a function of the proportion of the vehicle mass on the tractive axle. Typical axle mass distributions for different truck types were presented by Rakha *et al*. (2001).

Rakha and Lucic (2002) introduced the $\beta$ factor into Equation 3 in order to account for the gear-shifting impacts at low truck speeds. While the variable power factor does not incorporate gear shifting explicitly, it does account for the major behavioral characteristics that result from gear shifting, namely the reductions of power as gearshifts are being engaged. Specifically, the factor is a linear relation of vehicle speed with an intercept of $1/v_0$ and a maximum value of 1.0 at a speed $v_0$ (optimum speed or the speed at which the vehicle attains its full power), as demonstrated in Equation 6. The intercept guarantees that the vehicle has enough power to accelerate from a complete stop. The calibration of the variable power factor was conducted by experimenting with different truck and weight combinations to estimate the speed at which the vehicle power reaches its maximum (termed the optimum speed). The optimum speed was found to vary as a function of the weight-to-power ratio, as demonstrated in Equation 7. The details of how this relationship was derived are described by Rakha and Lucic (2002).

$$a = \frac{F - R}{M} \qquad [1]$$

$$\frac{d^2 x}{dt^2} = f(\frac{dx}{dt}, x) \qquad [2]$$

$$F_t = 3600\ \beta\, \eta\, \frac{P}{v} = \frac{K_T \beta}{v} \qquad [3]$$

$$F_{\max} = 9.8066\, M_{ta}\ \mu \qquad [4]$$

$$F = \min(F_t, F_{\max}) \qquad [5]$$

$$\beta = \frac{1}{v_0}\left[1 + \min(v, v_0)\left(1 - \frac{1}{v_0}\right)\right] \qquad [6]$$

$$v_0 = 1164 w^{-0.75} \qquad [7]$$

Three resistance forces are considered in the model (Mannering and Kilareski, 1998; Fitch, 1994; Archilla and De Cieza, 1999; Rakha *et al*., 2001). Namely, the aerodynamic resistance, or air drag, which is a function of the vehicle frontal area, the location altitude, the truck drag coefficient, and the square of speed of the truck, as indicated in Equations 8 and 9. The constant $c_1$ accounts for the air density at sea level at a temperature of 15ºC (59ºF). Typical values of vehicle frontal areas for different truck and bus types and typical drag coefficients are provided in the literature (Rakha *et al*., 2001).

The rolling resistance is a linear function of the vehicle speed and mass, as indicated in Equation 10. Typical values for rolling coefficients ($C_r$, $c_1$, *and* $c_2$), as a function of the road surface type, condition, and vehicle tires, are provided in the literature (Rakha *et al*., 2001). Generally, radial tires provide a resistance that is 25 percent less than that for bias ply tires.

The grade resistance is a constant that varies as a function of the vehicle's total mass and the percent grade that the vehicle travels along, as indicated in Equation 11. The grade resistance accounts for the proportion of the vehicle weight that resists the movement of the vehicle:

$$R_a = c_1 C_d C_h A v^2 = K_a v^2 \qquad [8]$$

$$C_h = 1 - 8.5 \times 10^{-5} H \qquad [9]$$

$$R_r = 9.8066\ C_r\ (c_2 v + c_3)\, \frac{M}{1000} = K_{r1} v + K_{r2} \qquad [10]$$

$$R_g = 9.8066\ M\ i \qquad [11]$$

## HCM EXAMPLE ILLUSTRATIONS

Having described the variable power vehicle dynamics model that was developed by Rakha and Lucic (2002), this section demonstrates how the model can be applied to a sample roadway section to estimate truck speeds. Furthermore, the model speed estimates are compared to speed estimates using the HCM truck performance curves for two reasons. First, the application demonstrates the consistency between the variable power vehicle dynamics model and the HCM procedures for similar truck and roadway characteristics. Second, by altering the truck and roadway parameters significant differences in truck behavior are observed and thus demonstrating the need to enhance and extend the HCM procedures. The procedures described by Rakha *et al*. (2001) to solve the ODE numerically are applied to the three examples that are presented.

### Example 1

A simple 3.2 km section of highway is considered for illustration purposes. The grades along the section include a 2 percent upgrade over 0.8 kilometers followed by a 5 percent upgrade over 0.8 kilometers followed by a 1 percent upgrade over the remainder of the section (length of 1.6 km).

Using the HCM truck performance curves that are illustrated in Figure 1 the speed of the truck is estimated at 0.1-km intervals, assuming an initial truck speed of 0 and 88 km/h, as illustrated in Figure 2. Figure 2 demonstrates that the HCM procedures estimate the final truck speed to be 81 km/h after traveling the entire 3.2-km test section. Similarly, considering an asphalt pavement that has a fair rating (PSI between 1.5 and 3.0) and radial tires (97 percent of the trucks in a survey conducted along I-81 (Rakha and Lucic, 2002)) the final speed produced by the variable power model is 83 km/h, resulting in a difference of less than 2.5 percent between the HCM procedures and the variable power vehicle dynamics model. Consequently, this example demonstrates the consistency between the variable power vehicle dynamics model and the HCM procedures for truck and pavement parameters similar to the HCM performance curves.

Figure 2 further demonstrates that the condition of the pavement can have a fairly significant impact on the truck acceleration behavior. Specifically, an asphalt pavement, which is classified as good (PSI greater than 3.0), results in a final speed of 87.3 km/h compared to the 83.0 km/h in the case of a fair asphalt surface, with a difference in the range of 5 percent. Alternatively, a poor asphalt surface (PSI less than 1.5) results in a reduction in the final speed from 83.0 km/h to 78.4 km/h. The ability to consider the effect of the pavement type and condition on truck acceleration behavior provides a unique application for the proposed variable power vehicle dynamics model.

Figure 3 demonstrates the impact of the truck weight-to-power ratio on the truck acceleration behavior for the section under consideration. Specifically, the truck final speed varies from 111.1 km/h to 66.7 km/h for a truck weight-to-power ratio of 60 versus 180 kg/kW, respectively. The sensitivity of the analysis to the design truck weight-to-power ratio not only highlights the shortcomings of the state-of-practice HCM truck performance curves, but furthermore, demonstrates the need for truck performance curves that can capture different roadway and vehicle characteristics.

### Example 2

To further compare the variable power vehicle dynamics model to the HCM procedures, a roadway section composed of a 2 percent upgrade over 1.5 kilometers followed by a 6 percent upgrade over 1.5 kilometers was analyzed (total section length of 3.0 km). The basic HCM truck performance curves were utilized to compute the truck speed after traveling the entire 3-kilometer roadway section. Similarly, the variable dynamics model was utilized to compute the truck's final speed, considering different pavement conditions. The results demonstrated that the HCM truck speed profile best matched the variable power vehicle dynamics model for a poor asphalt pavement surface (PSI less than 1.5). The results further demonstrated that the pavement condition has a significant impact on truck acceleration behavior. Specifically, the truck final speed varied from 38.9, to 37.3, to 36.0 km/h for a good, fair, and poor asphalt surface, respectively. The relative differences in comparison to the HCM final speed of 35.3 km/h ranged from 10.3, to 5.8, to 1.9 percent, respectively.

In addition, the speed difference along the study section between the variable power vehicle dynamics model for a good asphalt surface relative to the HCM procedures (designed for a fair or poor asphalt surface) was significant. Specifically, the speed difference for the acceleration scenario had a maximum error of 10 km/h, which is equivalent to a 20 percent difference.

## Example 3

The next example compares the HCM procedures to the Rakha and Lucic model predictions for sustained grade sections. Specifically, Figure 4 illustrates the variation in the equilibrium speed estimates using the HCM truck performance curves against the Rakha and Lucic model. The equilibrium speed is the maximum speed a vehicle may attain along a sustained grade section. This speed is computed as the speed that is attained when the vehicle has traveled a sufficiently long distance along the grade that the vehicle is unable to accelerate any further (acceleration equal to zero). By solving for the vehicle speed when the tractive force equals the summation of the resistance forces (vehicle acceleration equals zero), the equilibrium speed can be computed, as summarized in Equation 12 and 13 depending on whether the tractive force exceeds the maximum frictional force that can be sustained between the vehicle tires and the roadway surface. The derivation of Equations 12 and 13 are presented in Appendix A for the interested reader.

Specifically, in the case that $F_{max} \geq K_T\beta/v_0$ then Equation 12 is utilized.

$$v_m = -\frac{K_{r1}}{3K_a} + \frac{\sqrt[3]{2}(-K_{r1}{}^2 + 3K_a c)}{3K_a b} - \frac{b}{3\sqrt[3]{2}\cdot K_a} \qquad \forall \quad F_{\max} \geq \frac{K_T\beta}{v_0} \qquad [12]$$

Where:

$$b = \sqrt[3]{-27K_a^2 d - 2K_{r1}^3 - 9K_a K_{r1} c + \sqrt[2]{4(-K_{r1}^2 + 3K_a c)^3 + (-27K_a^2 d - 2K_{r1}^3 - 9K_a K_{r1} c)^2}}$$

$$c = \begin{cases} K_{r_2} + R_g & v_m \geq v_0 \\ K_{r_2} + R_g - \dfrac{K_T}{v_0} + \dfrac{K_T}{v_0^2} & v_m < v_0 \end{cases}$$

$$d = \begin{cases} -K_T & v_m \geq v_0 \\ -\dfrac{K_T}{v_0} & v_m < v_0 \end{cases}$$

Alternatively, in the case that $F_{max} < K_T\beta/v_0$ then Equation 13 applied.

$$v_m = \begin{cases} \dfrac{-K_{r1} + \sqrt{K_{r1}^2 - 4K_a\left(K_{r2} + R_g - F_{\max}\right)}}{2K_a} & \forall\, v_m \leq \dfrac{K_T\beta}{F_{\max}} \\ -\dfrac{K_{r1}}{3K_a} + \dfrac{\sqrt[3]{2}(-K_{r1}{}^2 + 3K_a f)}{3K_a e} - \dfrac{e}{3\sqrt[3]{2}\cdot K_a} & \forall\, v_m > \dfrac{K_T\beta}{F_{\max}} \end{cases} \qquad [13]$$

Where:

$$e = \sqrt[3]{27K_a^2K_T - 2K_{r1}^3 - 9K_aK_{r1}f + \sqrt[2]{4(-K_{r1}^2 + 3K_af)^3 + (27K_a^2K_T - 2K_{r1}^3 - 9K_aK_{r1}f)^2}}$$

$$f = K_{r_2} + R_g$$

Utilizing Equations 12 and 13 the equilibrium speed was computed considering a fair asphalt surface ($C_r$ = 1.75 and $\mu$ = 0.5), a truck weight-to-power ratio of 120 kg/kW (200 lb/hp), an engine power of 336 kW (450 hp), an engine efficiency of 88 percent, full aerodynamic features ($C_d$ = 0.58) with a frontal area of 10.7 $m^2$, and equipped with radial tires ($c_2$ = 0.0328 and $c_3$ = 4.575). The selected parameters reflect what has been documented in the literature as typical parameters (Rakha *et al.*, 2001) and reflective of in-field truck characteristics based on a survey conducted along I-81 in the state of Virginia (Rakha and Lucic, 2002).

Figure 4 compares the HCM equilibrium speed estimates against the Rakha and Lucic model equilibrium speed estimates for two scenarios. The first scenario incorporates the power reduction factor ($\beta$) to account for the loss of power during gear shifts at low truck speeds while the second formulation (dotted line) assumes that the truck can maintain its maximum power ($\beta$=1.0) at low speeds (i.e. $c = K_{r2}+R_g$ and $d = -K_T$ for the full range of vehicle speeds). The figure clearly demonstrates the consistency between the HCM truck performance curves and the Rakha and Lucic model estimates for the full range of grades, except for a grade of 8 percent. Specifically, for a grade of 8 percent the Rakha and Lucic model estimates an equilibrium speed that is lower than what the HCM procedures and what the constant power model would suggest. The reason the variable power model estimates a lower equilibrium speed is because the equilibrium speed is less than the optimum speed of 32 km/h ($v_0$), and thus the values of the constants *c* and *d* take the second form of Equation 12 (i.e. $c = K_{r2}+R_g-K_T/v_0+K_T/v_0^2$ and $d = -K_T/v_0$). Field observations have demonstrated that vehicles are unable to attain the equilibrium speeds that are proposed by a constant power model, as was discussed in detail by Rakha and Lucic (2002). Consequently, the HCM procedures would tend to over-estimate truck equilibrium speeds when the equilibrium speed is less than the optimum speed.

### Example Conclusions

Based on the comparisons that were presented in Examples 1, 2, and 3, it appears that the HCM truck performance curves were developed for an asphalt surface that ranges between fair and poor. Furthermore, the examples clearly demonstrate a reasonable degree of consistency between the HCM procedures and model predictions for similar truck and roadway characteristics. Example 3 demonstrates the inability of the HCM procedures to capture the power losses at low speeds when a truck in engaged in gear-shifting. Finally, the examples also demonstrate the ineffectiveness of the HCM truck performance curves to reflect different pavement and truck characteristics that have a significant impact on truck performance along grade sections.

## TRUCKSIM SOFTWARE OVERVIEW

### Model Structure

The TRUCKSIM software is a computer program that utilizes the previously described variable power vehicle dynamics model to simulate the motion of a truck along a roadway section. Specifically, the program solves the second order ODE that is presented in Equation 14 by recasting the model as a system of two first-order equations (an $n^{th}$-order equation reduces to a set of n $1^{st}$-order equations), as demonstrated in Equation 15.These ODEs are then solved using a first-order Euler approximation, as demonstrated in Equations 15 and 16. Specifically, Equations

16 and 17 are solved numerically by simulating the motion of the truck at small time steps (Δt). The program updates the vehicle's speed and position in a stepwise fashion by computing the vehicle's speed and position at each instant $t_i$ based on its speed and position at instant $t_{i-1}$. The user specified solution step size is a parameter that affects the accuracy of the truck modeling. For all the examples that are illustrated in this paper a deci-second time step size was selected. Finally, the program execution stops when one of a number of criteria is achieved. For example, a criterion could be to achieve the truck equilibrium speed (also known as the crawl speed) or the model may be executed to simulate the motion of a vehicle along a specific roadway section of a given length.

$$a(t_i) = \frac{F(t_i) - R(t_i)}{M} \qquad [14]$$

$$\begin{Bmatrix} \dfrac{d(v(t_i))}{dt} \\ \dfrac{d(x(t_i))}{dt} \end{Bmatrix} = \begin{Bmatrix} a(t_i) \\ v(t_i) \end{Bmatrix} \qquad [15]$$

$$v(t_i) = v(t_{i-1}) + a(t_{i-1})\Delta t \qquad [16]$$

$$x(t_i) = x(t_{i-1}) + v(t_{i-1})\Delta t \qquad [17]$$

### Model Input and Output

The program inputs include the pavement type and condition, the tire type, the altitude, the vehicle mass, vehicle power, percent mass on the tractive axle, presence of aerodynamic features, engine efficiency, and a user specified minimum speed for the design of climbing lanes. The pavement types include concrete, asphalt, macadam, and cobble. The pavement condition includes excellent, good, fair in the case of concrete pavements and good, fair, and poor in the case of asphalt pavements.

The program can provide three outputs. The first output is the vehicle's acceleration, speed, position, tractive force, and resistance forces at the user-specified time step for the duration of the simulation. The second output identifies the start and end locations of truck climbing lanes (locations where the truck speed decreases below a user-specified minimum speed). The final output of the program is the vehicle's equilibrium speed, which is computed by solving Equation 12, as was discussed earlier.

## DEVELOPMENT OF TRUCK PERFORMANCE CURVES

Having demonstrated the need to expand the HCM truck performance curves to cover a wider range of truck and pavement conditions, this section first evaluates the impact of truck weight, power, and weight-to-power ratio on truck performance curves. Subsequently, the impact of the roadway pavement type and condition on vehicle acceleration is analyzed. Finally, truck performance curves that are reflective of current truck and roadway conditions are developed.

Prior to developing the truck performance curves the basic input parameters that were utilized are described followed by a description of the parameters that were utilized to account for the various truck and pavement conditions that were studied.

## Basic Input Parameters

The basic input parameters reflected trucks equipped with full aerodynamic features given that these trucks were found to represent 55 percent of a 157 sample size that was gathered along I-81 in the state of Virginia (Rakha and Lucic, 2002). The assumption of full aerodynamic features implies the use of an aerodynamic coefficient of 0.58, as was described by Rakha *et al*., 2001. In addition, trucks were assumed to be equipped with radial tires (97 percent of the I-81 sample). The use of radial tires implies the incorporation of rolling resistance coefficients ($c_2$ and $c_3$) of 0.0328 and 4.575, respectively. In addition, the pavement surface was assumed to be a fair asphalt surface, implying a rolling resistance factor ($C_r$) equal to 1.75 and a coefficient of friction of 0.5. The engine efficiency was assumed to be 88 percent with an engine power of 336 kW (450 hp) (the mean power for the I-81 sample), and a vehicle weight-to-power ratio of 120 kg/kW (200 lb/hp). The altitude was assumed to be sea level ($C_h$ = 1.00). The truck frontal area was assumed to be 10.7 $m^2$ and the percentage mass on the tractive axle was assumed to be 35 percent, as recommended by Rakha *et al*., 2001. It should be noted that the vehicle engine efficiency may deteriorate with age. A characterization of engine efficiency as a function of engine age is beyond the scope of this paper; however is a subject worth further investigation.

## Impact of Truck Weight and Power on Truck Performance

The HCM truck performance curves utilize the vehicle weight-to-power ratio as the sole truck independent variable (input variable). This section investigates the effect of vehicle power on truck performance maintaining a constant vehicle weight-to-power ratio (i.e. simultaneously altering the vehicle weight and power while maintaining an identical weight-to-power ratio). Specifically, the power was varied between 224 kW (300 hp) to 485 kW (650 hp) at increments of 37.3 kW (50 hp). Table 1 summarizes the percentage change in the equilibrium speed relative to the base 350 hp scenario. In addition, Figure 5 illustrates the variation in the truck equilibrium speed as a function of the roadway grade and vehicle power for a constant weight-to-power ratio of 120 kg/kW. The results clearly indicate that the equilibrium speed varies as a function of the vehicle power even though the vehicle weight-to-power ratio remains constant. These differences in equilibrium speeds are more significant for mild grades (grades of 0 or 2 percent) with variations up to 24 percent and less significant differences at steeper roadway grades (differences less than 5 percent). Furthermore, the influence of the vehicle power on the truck performance is more significant for lower weight-to-power ratios in comparison to higher weight-to-power ratios. For example, differences in equilibrium speeds in the range of 24 percent are observed for weight-to-power ratios of 60 kg/kW traveling on a level roadway while these differences are reduced to 15 percent for weight-to-power ratios or 180 kg/kW.

This analysis demonstrates that it is reasonable to only consider the weight-to-power ratio (ignoring differences in vehicle power) for grades of 4 percent and higher. However, such a supposition would not be accurate for the modeling of trucks on level surfaces. Similar findings were observed for other pavement types and surface conditions. In conclusion, the accurate modeling of truck behavior considering a constant weight-to-power ratio depends on the distribution of the truck engine powers and roadway grades. A survey on I-81 concluded that over 60 percent of the truck engines were within the range between 336 and 373 kW (450 to 500 hp) (Rakha and Lucic, 2002). Consequently, differences in truck engine powers would appear to marginally affect the truck performance curves given the relatively narrow bandwidth of engine power variation.

Another important finding is that the weigh-to-power ratio significantly affects the truck equilibrium speed, as summarized in Table 2. Specifically, the equilibrium speed varies significantly between vehicle weight-to-power ratios of 60, 120, and 180 kg/kW regardless of the vehicle power. For example, the equilibrium speed drops by 32 km/h for a change in a vehicle weight-to-power ratio from 60 to 120 kg/kW and further drops by19 km/h for a change from 120 to 180 kg/kW, for travel on a 2 percent grade with an engine of 336 kW (450 hp). Consequently, it is recommended that truck performance curves be developed for different truck weight-to-power ratios.

## Impact of Pavement Type and Condition on Truck Performance

A number of pavement types and conditions were considered as part of the analysis. The pavement types included asphalt, concrete, macadam, cobble, and dirt roadways. Furthermore, the study investigated the impact of snow on truck acceleration behavior. The modeling of different pavement types and conditions was conducted by setting the $C_r$ and $\mu$ factors to reflect the pavement characteristics. The recommended values for these factors are described in detail in the literature (Rakha *et al*., 2001).

In addition to the pavement type, the pavement surface condition was also considered in the analysis. Specifically, three asphalt surface conditions were considered, namely good, fair, and poor condition. The pavement surface condition was characterized by the Pavement Serviceability Index (PSI), which is a measure of the quality of pavement surface. The Pavement Serviceability-Performance Concept was developed by Carey and Irick (1962) to handle the question concerning pavement failure. Carey and Irick considered pavement performance histories and noted that pavements usually deteriorate as traffic loading is applied in conjunction with prevailing environmental conditions. Studies have shown that new pavements have an initial PSI rating of approximately 4.2 to 4.5. The point at which a pavement is considered to have failed is termed the Terminal Serviceability Index (TSI). The TSI is highway dependent and ranges between 2.5 and 3.0 for interstate highways and principal arterials.

Three concrete pavement conditions were considered in the analysis. These conditions include excellent, good, and fair surface. An excellent concrete surface is reflective of a new rigid pavement surface without expansion cracks. A fair concrete surface is characterized by a pavement that does include tracks and does offer a fairly uncomfortable ride. Finally, a poor concrete surface is a surface that has significant surface cracks and defects that offers considerable discomfort in terms of vehicle rideability.

Figure 6 illustrates how the truck equilibrium speed varies as a function of the truck weight-to-power ratio, roadway grade, pavement type, and pavement condition. Figure 6 clearly demonstrates, as would be expected, that the truck equilibrium speed decreases as the roadway grade and truck weight-to-power ratio increases, as was demonstrated earlier in Table 2. Furthermore, the figure illustrates that the effect of pavement type and condition is more pronounced for higher truck weight-to-power ratios traveling on lower roadway grades (less than 3 percent grade). Figure 6 demonstrates that in general concrete pavements provide better acceleration behavior than asphalt pavements. Furthermore, 5 cm (2 inch) and 10 cm (4 inch) snow result in a significant reduction in vehicle acceleration capabilities especially along steep grade sections (grades greater than 4 percent).

## Truck Performance Curves

Having demonstrated the significant impact of the truck weight-to-power ratio, pavement type, and pavement condition on truck acceleration behavior, Figure 8 through Figure 10 develop truck performance curves for various truck and pavement characteristics. Specifically, Figure 8, Figure 9, and Figure 10 illustrate the variation in truck performance curves for different pavement types, pavement conditions, and for three truck weight-to-power ratios including 60, 90, and 120 kg/kW, respectively. The truck performance curves that are illustrated in Figure 8 through Figure 10 are intended to replace the single truck performance curve that is presented in the HCM and the AASHTO Geometric Design Guide for the design of climbing lanes along major highways. It should be noted that the curves assume that truck speeds are only constrained by the vehicle dynamics and thus may not reflect speed limit effects on vehicle speeds. However, it should be noted that accounting for the speed limit is easily achieved by considering a maximum vehicle speed in modeling truck behavior.

## EXAMPLE APPLICATION OF MODEL

In order to demonstrate the potential benefits of the TRUCKSIM software, the software was run on a 45-km section of I-81. Specifically, a 45-km section of I-81 in the state of Virginia from milepost 118 to milepost 143 between Christiansburg and Roanoke was tested as part of this research effort, as illustrated Figure 11. The southbound traffic travel upgrade (from left to right), while the northbound traffic travel downgrade (from right to left). The vertical profiles for both directions are similar in many aspects except for an exceptionally high upgrade in the southbound direction in the Christiansburg exit vicinity.

Figure 12 illustrates the spatial variation in truck speed for a 120 kg/kW truck traveling in the southbound direction. The northbound speed profile is not illustrated, however the results are similar. Specifically, the results indicate that a reasonable proportion of the section results in drops in truck speeds below the speed limit of 104 km/h (65 mph) for a good asphalt surface. Specifically, 34 percent of the 45 km section (approximately 15.4 km) involves trucks traveling below the 104 km/h speed limit in the case of a good asphalt surface (good asphalt surface with radial tires), as demonstrated in Table 3. The percentage of highway length with speeds less than 104 km/h increases from 34 percent in the case of a good asphalt surface to 44 percent in the case of a fair asphalt surface and finally to 54 percent in the case of a poor asphalt surface. If a heavier truck is utilized for design purposes (weight-to-power ratio of 150 kg/kW) then a higher percentage of the roadway length would require climbing lanes. Specifically, the percentage length of roadway section requiring climbing lanes varies from 43, to 52, to 67 percent, for a good, fair, and poor asphalt surface, respectively. In the case of a concrete pavement surface climbing lanes are required along 27, 39, and 47 percent of the 45-km roadway section considering a 120 kg/kW truck equipped with radial tires. Consequently, Table 3 demonstrates that the use of a concrete pavement surface as opposed to an asphalt surface can result in a 5 to 10 percent reduction in the climbing lane requirements. The location of the climbing lanes along the 45-km section are demonstrated in Figure 12. Furthermore, Figure 12 clearly demonstrates the impact of the pavement condition on vehicle acceleration behavior.

Table 3 also demonstrates the effect of snow on truck climbing lane requirements. It is interesting to note that in some rare instances the increase in the truck weight-to-power ratio results in a reduction in the climbing lane requirements (for a 4-inch snow surface). This reduction in climbing lane requirements is caused by the fact that the very low coefficient of friction results in a small $F_{max}$, which can be increased by increasing the mass on the tractive axle. Consequently, by increasing the mass of the truck the vehicle acceleration behavior is enhanced until the $F_{max}$ exceeds the tractive force ($F_T$).

A comparison of Table 3 and Table 4 demonstrates that by increasing the desired minimum speed the percentage of roadway requiring a climbing lane increases considerably. Specifically, by increasing the minimum desired speed from the speed limit of 104 km/h to the design speed of 112 km/h, the percentage roadway requiring a climbing lane increases from 34 to 44 percent in the southbound direction of travel for a good asphalt surface considering a truck equipped with radial tires. Similarly, the percentage of roadway requiring a climbing lane increases from 54 to 70 percent in the case of a poor asphalt surface.

As was evident from the roadway vertical profile that was demonstrated earlier in Figure 11 that travel along the southbound direction involves moving along more significant and sustained upgrade sections. Consequently, only 6, 17, and 33 percent of the 45-km northbound section involves travel at speeds less than the 104 km/h speed limit as a result of vehicle dynamics limitations for a good, fair, and poor asphalt surface, respectively, as demonstrated in Table 5.

## CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The paper demonstrates that the variable power vehicle dynamics model developed by Rakha and Lucic (2002) produces truck acceleration behavior that is consistent with the HCM and AASHTO design procedures if identical truck and roadway characteristics are incorporated. The model offers a number of advantages over the HCM and AASHTO procedures given that it is sensitive to truck, roadway, and pavement characteristics. The paper develops the TRUCKSIM software that solves the second order ODE for estimating the speed profile of a truck along a composite grade section. The software can assist practitioners in identifying locations of climbing lanes along roadway segments. Furthermore, using the software, the paper extends the HCM and AASHTO performance curves to cover different truck weight-to-power ratios, different pavement types, and different pavement conditions. The paper demonstrates that the vehicle weight-to-power ratio is a critical variable in designing climbing lanes. Finally, the paper also demonstrates that the vehicle power, in addition to the vehicle weight-to-power ratio, are critical variables in the design of truck climbing especially along level and mild upgrade sections.

It should be noted that the engine efficiency may deteriorate with the vehicle and engine age. The study does not attempt to quantify the effect of vehicle age on the engine efficiency; however it is recommended that further research be conducted in order to characterize the deterioration of engine efficiency as a function of engine age and its impact on the design of truck climbing lanes.

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## VARIABLE DEFINITIONS

$A$ = Vehicle frontal area (m$^2$)
$\eta$ = Transmission efficiency
$\mu$ = Coefficient of friction between tires and pavement
$a$ = The maximum vehicle acceleration (m/s$^2$)
$a(t_i)$ = Vehicle acceleration at instant $t_i$
$C_d$ = Vehicle drag coefficient
$C_h$ = Altitude coefficient
$C_r$ = Rolling coefficient
$c_1$ = Constant (0.047285)
$c_2$, $c_3$ = Rolling resistance coefficients
$F$ = Tractive effort effectively acting on truck (N)
$F(t_i)$ = Effective tractive force at instant $t_i$
$F_{max}$ = Maximum tractive force
$F_t$ = Tractive effort (N)
$H$ = Altitude (m)
$i$ = Percent grade (m/100 m)
$M$ = Vehicle mass (kg)
$M_{ta}$ = Vehicle mass on tractive axle (kg)
$P$ = Engine power (kW)
$R$ = Total resistance force, which is the sum of the aerodynamic, rolling, and grade resistance forces (N)
$R_a$ = Air drag or aerodynamic resistance (N)
$R_g$ = Grade resistance (N)
$R_r$ = Rolling resistance (N)
$v$ = Vehicle speed (km/h)
$v(t_i)$ = Vehicle speed at instant $t_i$
$v_0$ = Optimum speed which is the speed at which a vehicle attains maximum power (km/h)
$w$ = Vehicle weight-to-power ratio (kg/kW)
$x$ = Distance traveled by vehicle (m)
$x(t_i)$ = Vehicle location along test section at instant $t_i$.
$\Delta t$ = Duration of time interval used for solving the ODE (in this case 1-second duration)
$\beta$ = Vehicle power reduction factor (unitless)
$v_m$ = Equilibrium speed of a vehicle (km/h)
$K_T$ = Tractive force constant (kW). $K_T = 3600\eta P$
$K_a$ = Aerodynamic resistance force constant. $K_a = c_1 C_d C_h A$
$K_{r1}$ = Rolling resistance force speed coefficient. $K_{r_1} = \dfrac{9.8066 \cdot C_r \cdot M}{1000} \cdot c_2$
$K_{r2}$ = Rolling resistance force constant. $K_{r_2} = \dfrac{9.8066 \cdot C_r \cdot M}{1000} \cdot c_3$
$b$ = $\sqrt[3]{-27K_a^2 d - 2K_{r1}^3 - 9K_a K_{r1} c + \sqrt[2]{4(-K_{r1}^2 + 3K_a c)^3 + (-27K_a^2 d - 2K_{r1}^3 - 9K_a K_{r1} c)^2}}$

$$c = \begin{cases} K_{r_2} + R_g & v_m \geq v_0 \\ K_{r_2} + R_g - \dfrac{K_T}{v_0} + \dfrac{K_T}{v_0^2} & v_m < v_0 \end{cases}$$

$$d = \begin{cases} -K_T & v_m \geq v_0 \\ -\dfrac{K_T}{v_0} & v_m < v_0 \end{cases}$$

$$e = \sqrt[3]{27K_a^2K_T - 2K_{r1}^3 - 9K_aK_{r1}f + \sqrt[2]{4(-K_{r1}^2 + 3K_af)^3 + (27K_a^2K_T - 2K_{r1}^3 - 9K_aK_{r1}f)^2}}$$

$$f = K_{r_2} + R_g$$

## LIST OF TABLES

## LIST OF FIGURES

## Table 1: Percentage Change in Equilibrium Speed as a Function of Truck Power

| | | 60 kg/kW (100 lb/hp) | | | | | | | | 120 kg/kW (200 lb/hp) | | | | | | | | 180 kg/kW (300 lb/hp) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 223 | 261 | 298 | 336 | 373 | 410 | 448 | 485 | 223 | 261 | 298 | 336 | 373 | 410 | 448 | 485 | 223 | 261 | 298 | 336 | 373 | 410 | 448 | 485 |
| Asphalt Goo | 0 | 0% | 5% | 9% | 12% | 16% | 19% | 21% | 24% | 0% | 4% | 7% | 10% | 13% | 15% | 17% | 19% | 0% | 3% | 6% | 8% | 10% | 12% | 14% | 15% |
| | 2 | 0% | 3% | 6% | 9% | 11% | 13% | 15% | 17% | 0% | 2% | 3% | 5% | 6% | 7% | 7% | 8% | 0% | 1% | 2% | 2% | 3% | 3% | 4% | 4% |
| | 4 | 0% | 2% | 4% | 6% | 7% | 9% | 10% | 11% | 0% | 1% | 1% | 2% | 2% | 2% | 3% | 3% | 0% | 0% | 0% | 0% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 1% | 3% | 4% | 4% | 5% | 6% | 6% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 1% | 2% | 2% | 3% | 3% | 3% | 4% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 1% | 1% | 1% | 1% | 1% |
| Asphalt Fair | 0 | 0% | 4% | 8% | 11% | 14% | 17% | 20% | 22% | 0% | 3% | 6% | 8% | 11% | 13% | 14% | 16% | 0% | 2% | 5% | 6% | 8% | 9% | 10% | 12% |
| | 2 | 0% | 3% | 6% | 8% | 10% | 12% | 14% | 15% | 0% | 1% | 3% | 4% | 5% | 5% | 6% | 7% | 0% | 1% | 1% | 2% | 2% | 3% | 3% | 3% |
| | 4 | 0% | 2% | 4% | 5% | 7% | 8% | 9% | 10% | 0% | 1% | 1% | 1% | 2% | 2% | 2% | 2% | 0% | 0% | 0% | 1% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 1% | 2% | 3% | 4% | 5% | 5% | 6% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 1% | 1% | 2% | 2% | 3% | 3% | 3% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Asphalt Poor | 0 | 0% | 4% | 7% | 10% | 13% | 16% | 18% | 20% | 0% | 3% | 5% | 7% | 9% | 11% | 12% | 13% | 0% | 2% | 4% | 5% | 6% | 7% | 8% | 9% |
| | 2 | 0% | 3% | 5% | 8% | 9% | 11% | 13% | 14% | 0% | 1% | 2% | 3% | 4% | 4% | 5% | 5% | 0% | 1% | 1% | 1% | 2% | 2% | 2% | 2% |
| | 4 | 0% | 2% | 4% | 5% | 6% | 7% | 8% | 9% | 0% | 0% | 1% | 1% | 1% | 2% | 2% | 2% | 0% | 0% | 0% | 1% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 1% | 2% | 3% | 4% | 4% | 5% | 5% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 5% | 7% | 8% | 8% | 8% | 9% | 9% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Concrete Exc | 0 | 0% | 5% | 9% | 13% | 16% | 19% | 22% | 25% | 0% | 4% | 8% | 11% | 14% | 16% | 19% | 21% | 0% | 3% | 7% | 9% | 12% | 14% | 16% | 17% |
| | 2 | 0% | 4% | 7% | 9% | 12% | 14% | 16% | 18% | 0% | 2% | 4% | 5% | 6% | 7% | 8% | 9% | 0% | 1% | 2% | 3% | 3% | 4% | 4% | 4% |
| | 4 | 0% | 2% | 4% | 6% | 8% | 9% | 10% | 11% | 0% | 1% | 1% | 2% | 2% | 3% | 3% | 3% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 2% | 3% | 4% | 5% | 5% | 6% | 7% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 1% | 2% | 2% | 3% | 3% | 4% | 4% | 0% | 0% | 0% | 1% | 1% | 1% | 2% | 2% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Concrete Goo | 0 | 0% | 4% | 8% | 12% | 15% | 18% | 20% | 23% | 0% | 3% | 7% | 9% | 12% | 14% | 16% | 17% | 0% | 3% | 5% | 7% | 9% | 11% | 12% | 13% |
| | 2 | 0% | 3% | 6% | 9% | 11% | 13% | 14% | 16% | 0% | 2% | 3% | 4% | 5% | 6% | 7% | 7% | 0% | 1% | 2% | 2% | 2% | 3% | 3% | 3% |
| | 4 | 0% | 2% | 4% | 6% | 7% | 8% | 9% | 10% | 0% | 1% | 1% | 2% | 2% | 2% | 2% | 3% | 0% | 0% | 0% | 1% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 1% | 3% | 4% | 4% | 5% | 5% | 6% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 1% | 2% | 2% | 3% | 3% | 3% | 4% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Concrete Po | 0 | 0% | 4% | 8% | 11% | 14% | 16% | 19% | 21% | 0% | 3% | 6% | 8% | 10% | 12% | 13% | 14% | 0% | 2% | 4% | 6% | 7% | 8% | 9% | 10% |
| | 2 | 0% | 3% | 6% | 8% | 10% | 12% | 13% | 15% | 0% | 1% | 2% | 3% | 4% | 5% | 5% | 6% | 0% | 1% | 1% | 2% | 2% | 2% | 2% | 3% |
| | 4 | 0% | 2% | 4% | 5% | 6% | 7% | 8% | 9% | 0% | 0% | 1% | 1% | 2% | 2% | 2% | 2% | 0% | 0% | 0% | 1% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 1% | 2% | 3% | 4% | 4% | 5% | 5% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 1% | 1% | 2% | 2% | 3% | 3% | 3% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Snow 2" | 0 | 0% | 4% | 7% | 10% | 13% | 15% | 17% | 19% | 0% | 2% | 5% | 7% | 8% | 10% | 11% | 12% | 0% | 2% | 3% | 4% | 5% | 6% | 7% | 8% |
| | 2 | 0% | 3% | 5% | 7% | 9% | 10% | 12% | 13% | 0% | 1% | 2% | 3% | 4% | 4% | 5% | 5% | 0% | 1% | 1% | 1% | 2% | 2% | 2% | 2% |
| | 4 | 0% | 3% | 4% | 5% | 6% | 8% | 8% | 9% | 0% | 0% | 1% | 1% | 1% | 2% | 2% | 2% | 0% | 0% | 0% | 1% | 1% | 1% | 1% | 1% |
| | 6 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Snow 4" | 0 | 0% | 3% | 6% | 8% | 10% | 12% | 14% | 15% | 0% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 0% | 1% | 2% | 3% | 3% | 4% | 4% | 4% |
| | 2 | 0% | 5% | 10% | 14% | 18% | 21% | 24% | 27% | 0% | 1% | 1% | 2% | 2% | 3% | 3% | 3% | 0% | 0% | 1% | 1% | 1% | 1% | 1% | 1% |
| | 4 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 6 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 8 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |

## Table 2: Variation in Equilibrium Speed as a Function of Truck Weight-to-Power Ratio

| | | 60 kg/kW (100 lb/hp) | | | | | | | | 120 kg/kW (200 lb/hp) | | | | | | | | 180 kg/kW (300 lb/hp) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 223 | 261 | 298 | 336 | 373 | 410 | 448 | 485 | 223 | 261 | 298 | 336 | 373 | 410 | 448 | 485 | 223 | 261 | 298 | 336 | 373 | 410 | 448 | 485 |
| Asphalt Goo | 0 | 122 | 128 | 132 | 137 | 141 | 145 | 148 | 151 | 111 | 115 | 119 | 122 | 125 | 128 | 130 | 132 | 102 | 105 | 107 | 110 | 112 | 114 | 115 | 117 |
| | 2 | 101 | 105 | 108 | 110 | 113 | 115 | 117 | 119 | 76 | 77 | 78 | 79 | 80 | 81 | 81 | 82 | 58 | 59 | 59 | 59 | 60 | 60 | 60 | 60 |
| | 4 | 83 | 85 | 86 | 88 | 89 | 90 | 91 | 92 | 52 | 53 | 53 | 53 | 53 | 54 | 54 | 54 | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 37 |
| | 6 | 68 | 69 | 69 | 70 | 71 | 71 | 72 | 72 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 26 | 26 | 26 | 26 | 26 | 26 | 26 | 26 |
| | 8 | 56 | 56 | 57 | 57 | 57 | 58 | 58 | 58 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 5 | 5 | 5 | 6 | 6 | 6 | 6 | 6 |
| Asphalt Fair | 0 | 118 | 123 | 127 | 131 | 134 | 138 | 141 | 143 | 103 | 107 | 110 | 112 | 114 | 116 | 118 | 120 | 91 | 94 | 96 | 97 | 99 | 100 | 101 | 102 |
| | 2 | 97 | 101 | 103 | 105 | 107 | 109 | 111 | 112 | 71 | 72 | 73 | 73 | 74 | 75 | 75 | 75 | 53 | 54 | 54 | 54 | 55 | 55 | 55 | 55 |
| | 4 | 80 | 81 | 83 | 84 | 85 | 86 | 87 | 87 | 50 | 50 | 50 | 50 | 51 | 51 | 51 | 51 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 |
| | 6 | 65 | 66 | 67 | 68 | 68 | 68 | 69 | 69 | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| | 8 | 54 | 55 | 55 | 56 | 56 | 56 | 56 | 56 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| Asphalt Poor | 0 | 113 | 118 | 122 | 125 | 128 | 131 | 134 | 136 | 96 | 99 | 101 | 103 | 105 | 106 | 108 | 109 | 83 | 84 | 86 | 87 | 88 | 89 | 89 | 90 |
| | 2 | 94 | 97 | 99 | 101 | 103 | 104 | 106 | 107 | 66 | 67 | 68 | 68 | 69 | 69 | 70 | 70 | 50 | 50 | 50 | 50 | 50 | 50 | 51 | 51 |
| | 4 | 77 | 78 | 80 | 81 | 82 | 82 | 83 | 84 | 47 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 |
| | 6 | 63 | 64 | 65 | 65 | 66 | 66 | 66 | 67 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| | 8 | 50 | 53 | 54 | 54 | 54 | 54 | 54 | 55 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |

**Table 3: I-81 Southbound Section Climbing Lane Requirements (Minimum Desired = 104 km/h (65 mph))**

| Pavement | Weight-to-Power Ratio (kg/kW) -- Bias Ply Tires | | | | | | Weight-to-Power Ratio (kg/kW) -- Radial Tires | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 30 | 60 | 90 | 120 | 150 | 180 | 30 | 60 | 90 | 120 | 150 | 180 |
| Concrete Excellent | 0% | 12% | 24% | 36% | 44% | 48% | 0% | 10% | 21% | 27% | 36% | 43% |
| Concrete Good | 0% | 19% | 36% | 47% | 60% | 69% | 0% | 14% | 27% | 39% | 46% | 52% |
| Concrete Poor | 0% | 27% | 48% | 66% | 79% | 87% | 0% | 19% | 36% | 47% | 59% | 69% |
| Asphalt Good | 0% | 16% | 30% | 43% | 49% | 59% | 0% | 11% | 23% | 34% | 43% | 47% |
| Asphalt Fair | 0% | 22% | 42% | 57% | 69% | 78% | 0% | 17% | 32% | 44% | 52% | 61% |
| Asphalt Poor | 0% | 32% | 56% | 73% | 85% | 90% | 0% | 21% | 41% | 54% | 67% | 76% |
| Snow 2" | 33% | 36% | 63% | 81% | 89% | 92% | 20% | 23% | 44% | 61% | 74% | 83% |
| Snow 4" | 83% | 74% | 83% | 85% | 77% | 66% | 81% | 48% | 75% | 88% | 84% | 85% |

**Table 4: I-81 Southbound Section Climbing Lane Requirements (Minimum Desired = 112 km/h (70 mph))**

| Pavement | Weight-to-Power Ratio (kg/kW) -- Bias Ply Tires | | | | | | Weight-to-Power Ratio (kg/kW) -- Radial Tires | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 30 | 60 | 90 | 120 | 150 | 180 | 30 | 60 | 90 | 120 | 150 | 180 |
| Concrete Excellent | 0% | 19% | 34% | 45% | 51% | 61% | 0% | 16% | 26% | 38% | 45% | 49% |
| Concrete Good | 0% | 30% | 46% | 62% | 75% | 85% | 0% | 20% | 38% | 47% | 57% | 67% |
| Concrete Poor | 10% | 39% | 63% | 81% | 90% | 93% | 0% | 30% | 46% | 62% | 75% | 85% |
| Asphalt Good | 0% | 22% | 41% | 51% | 63% | 73% | 0% | 18% | 32% | 44% | 49% | 56% |
| Asphalt Fair | 6% | 34% | 54% | 72% | 84% | 90% | 0% | 23% | 42% | 53% | 67% | 76% |
| Asphalt Poor | 11% | 45% | 72% | 87% | 93% | 96% | 3% | 33% | 52% | 70% | 83% | 89% |
| Snow 2" | 44% | 51% | 79% | 91% | 96% | 97% | 31% | 36% | 59% | 78% | 88% | 91% |
| Snow 4" | 87% | 90% | 88% | 88% | 78% | 67% | 94% | 64% | 86% | 95% | 89% | 89% |

**Table 5: I-81 Northbound Section Climbing Lane Requirements (Minimum Desired = 104 km/h (65 mph))**

| Pavement | Weight-to-Power Ratio (kg/kW) -- Bias Ply Tires | | | | | | Weight-to-Power Ratio (kg/kW) -- Radial Tires | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 30 | 60 | 90 | 120 | 150 | 180 | 30 | 60 | 90 | 120 | 150 | 180 |
| Concrete Excellent | 0% | 1% | 2% | 7% | 13% | 21% | 0% | 0% | 0% | 1% | 5% | 7% |
| Concrete Good | 0% | 2% | 11% | 24% | 39% | 51% | 0% | 1% | 3% | 10% | 19% | 29% |
| Concrete Poor | 0% | 7% | 24% | 49% | 60% | 67% | 0% | 2% | 11% | 24% | 39% | 51% |
| Asphalt Good | 0% | 1% | 6% | 15% | 25% | 37% | 0% | 0% | 1% | 6% | 10% | 17% |
| Asphalt Fair | 0% | 4% | 18% | 36% | 52% | 60% | 0% | 1% | 7% | 17% | 29% | 39% |
| Asphalt Poor | 0% | 11% | 36% | 57% | 67% | 72% | 0% | 3% | 16% | 33% | 50% | 58% |
| Snow 2" | 10% | 16% | 48% | 63% | 72% | 78% | 3% | 6% | 21% | 44% | 56% | 64% |
| Snow 4" | 84% | 59% | 77% | 88% | 91% | 92% | 67% | 23% | 57% | 71% | 78% | 84% |

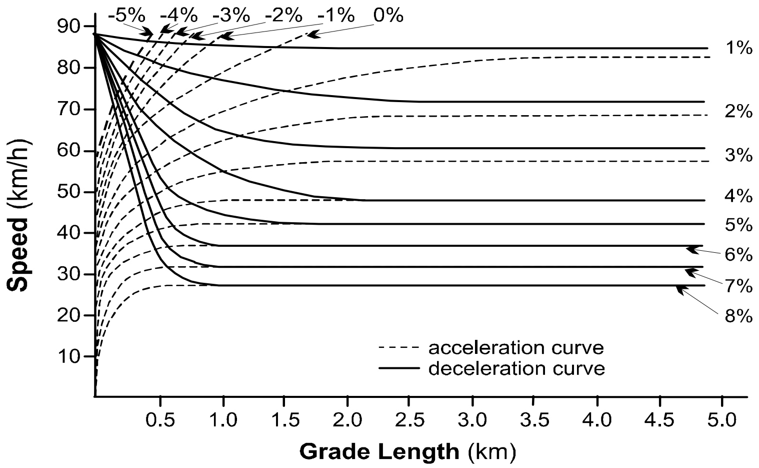
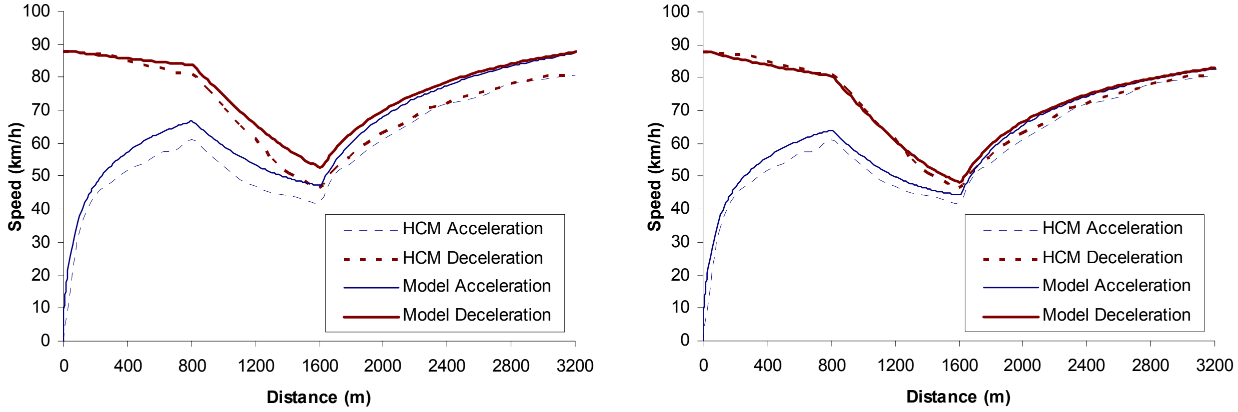


Figure 1: Highway Capacity Manual Truck Performance Curves (W/P = 120 kg/kW)


a. Good Asphalt – Radial Tires

b. Fair Asphalt – Radial Tires

c. Poor Asphalt – Radial Tires

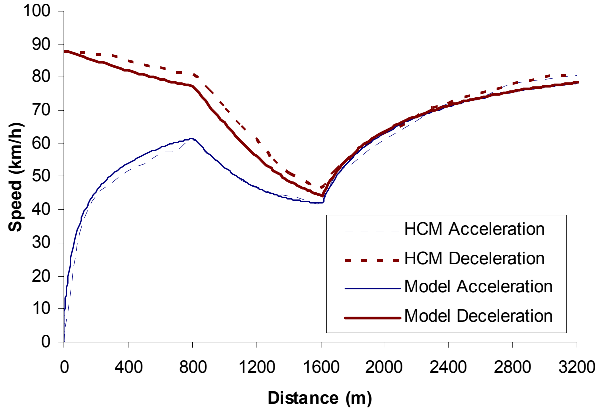
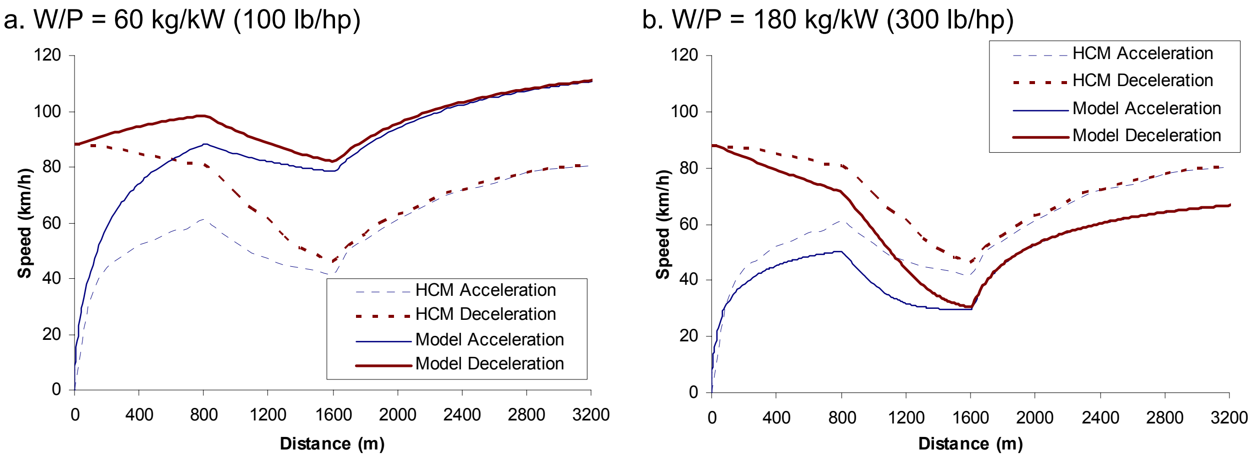


Figure 2: Truck Speed Profile (W/P = 120 kg/kW)


a. W/P = 60 kg/kW (100 lb/hp)

b. W/P = 180 kg/kW (300 lb/hp)

**Figure 3: Impact of Truck W/P Ratio on Truck Speed Profile**

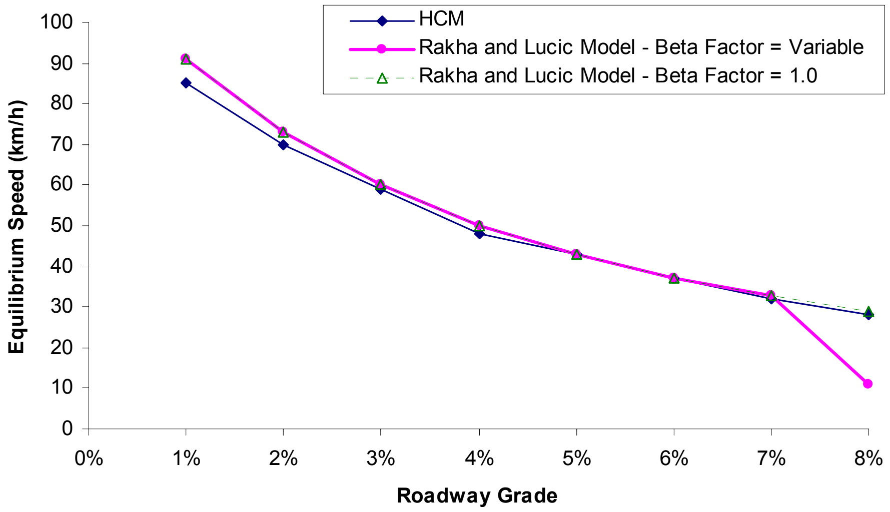


**Figure 4: Comparison of HCM and Model Equilibrium Speed Comparison**

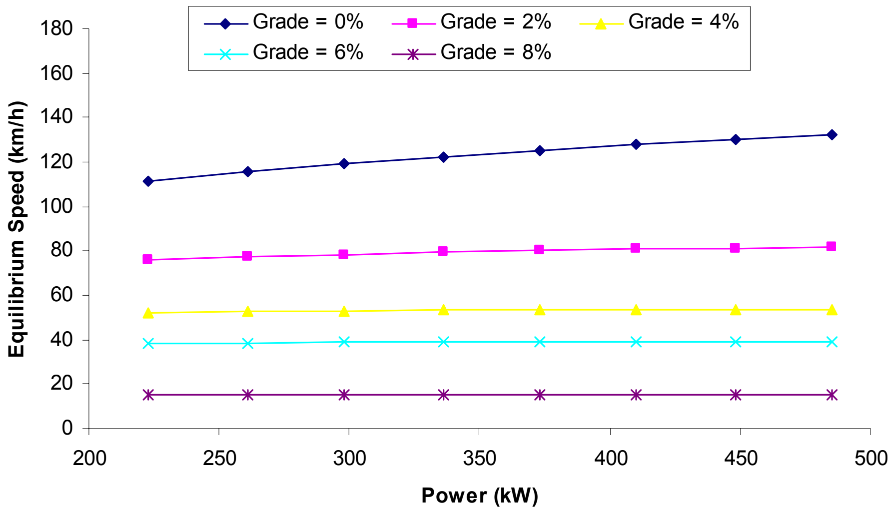


**Figure 5: Equilibrium Speed as a Function of Power and Road Grade (Weight-to-power ratio 120 kg/kW)**

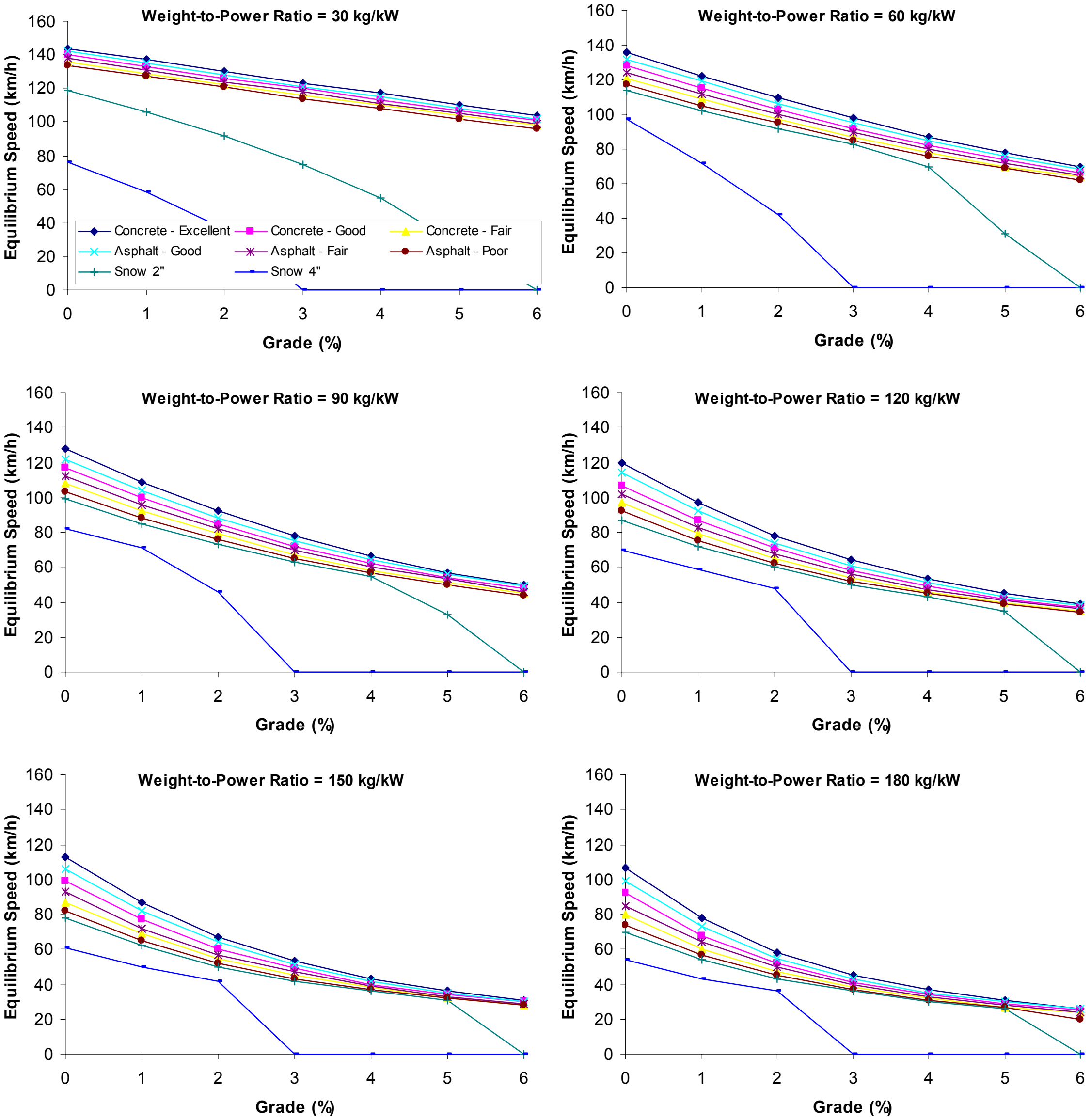


Figure 6: Truck Equilibrium Speed Variation as a Function of Roadway Grade and Vehicle W/P Ratio (Asphalt and Concrete Pavements)

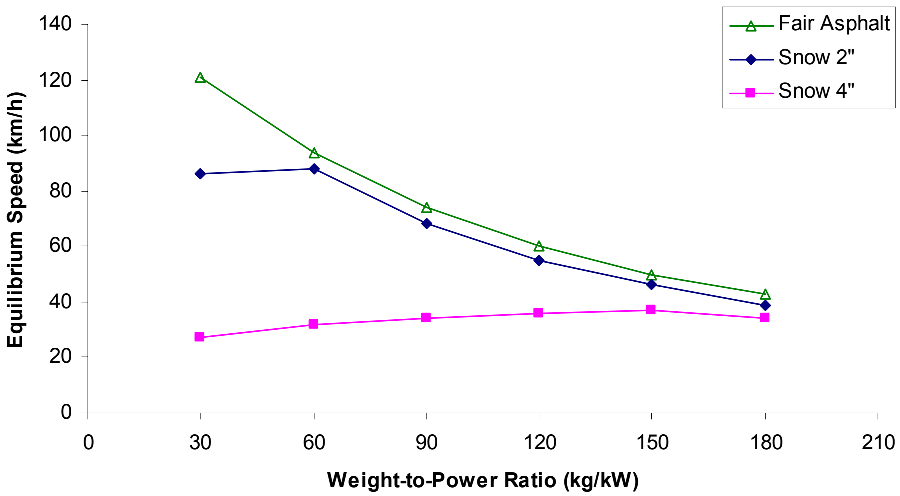


**Figure 7: Variation in Equilibrium Speed as a Function of Truck Weight-to-Power Ratio (Grade 3%)**

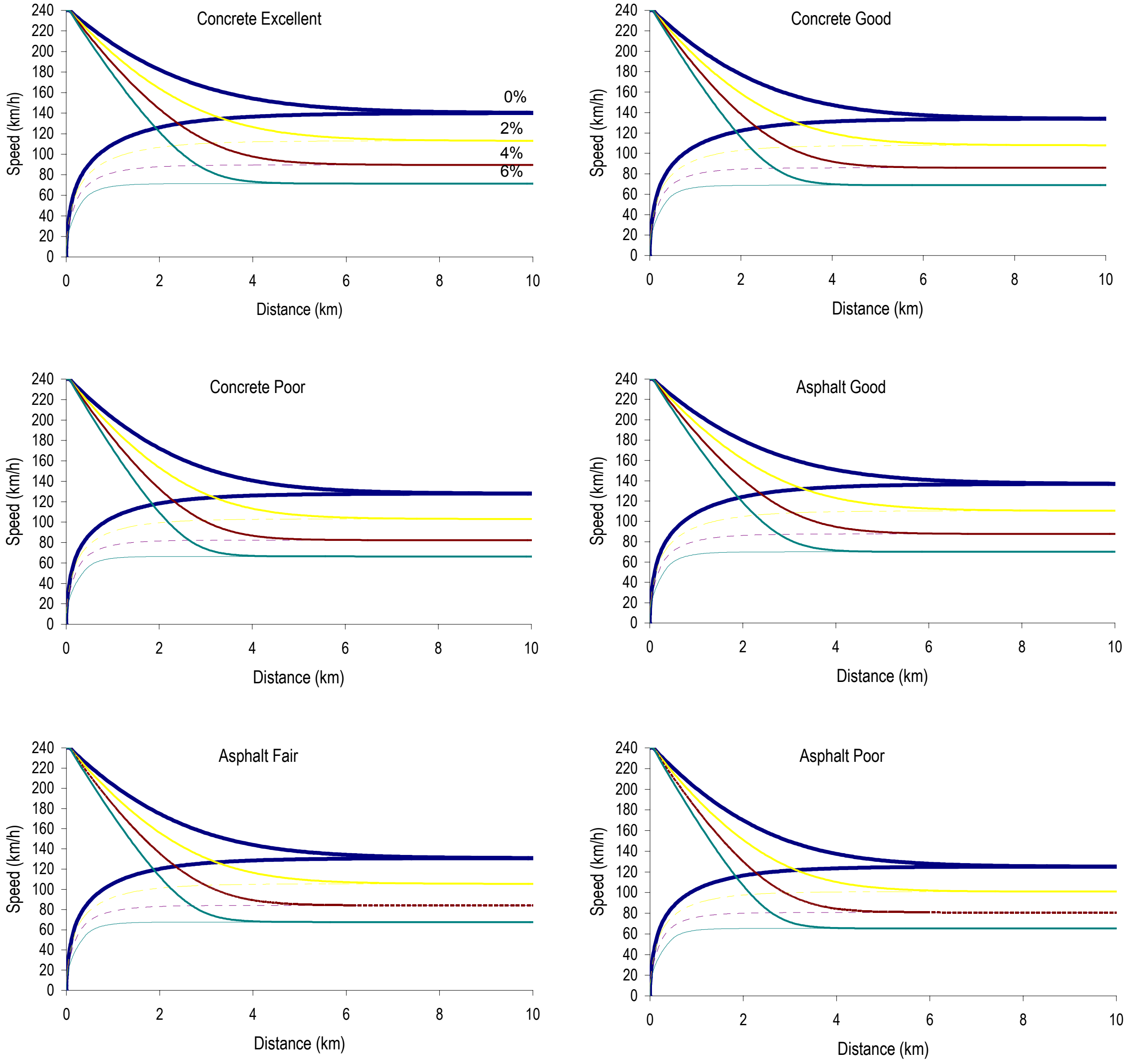


**Figure 8: Truck Performance Curve (Grade 0%, 2%, 4%, and 6%, Weight-to-Power Ratio 60 kg/kW, Power 335.7 kW)**

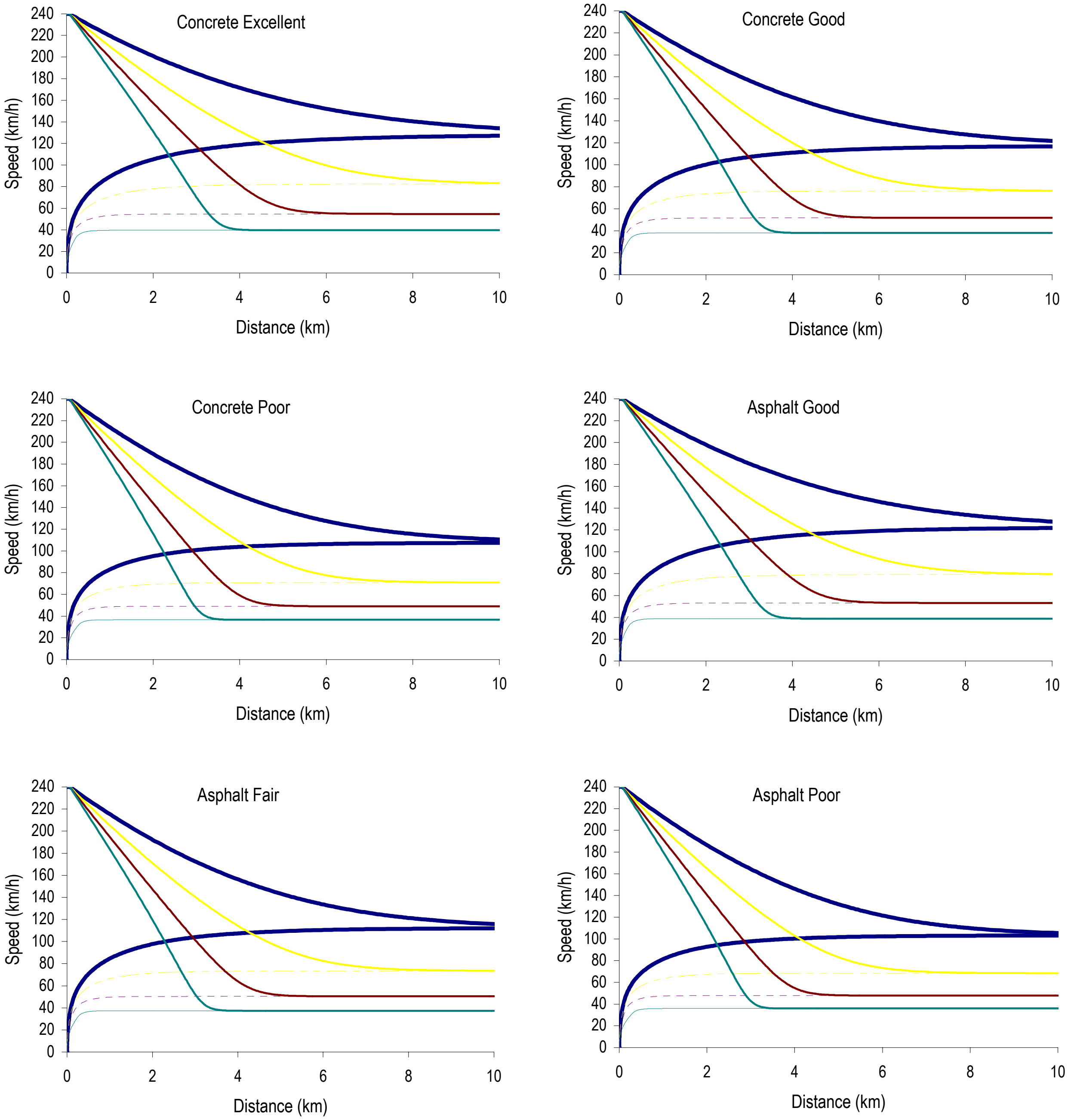


**Figure 9: Truck Performance Curve (Grade 0%, 2%, 4%, and 6%, Weight-to-Power Ratio 120 kg/kW, Power 335.7 kW)**

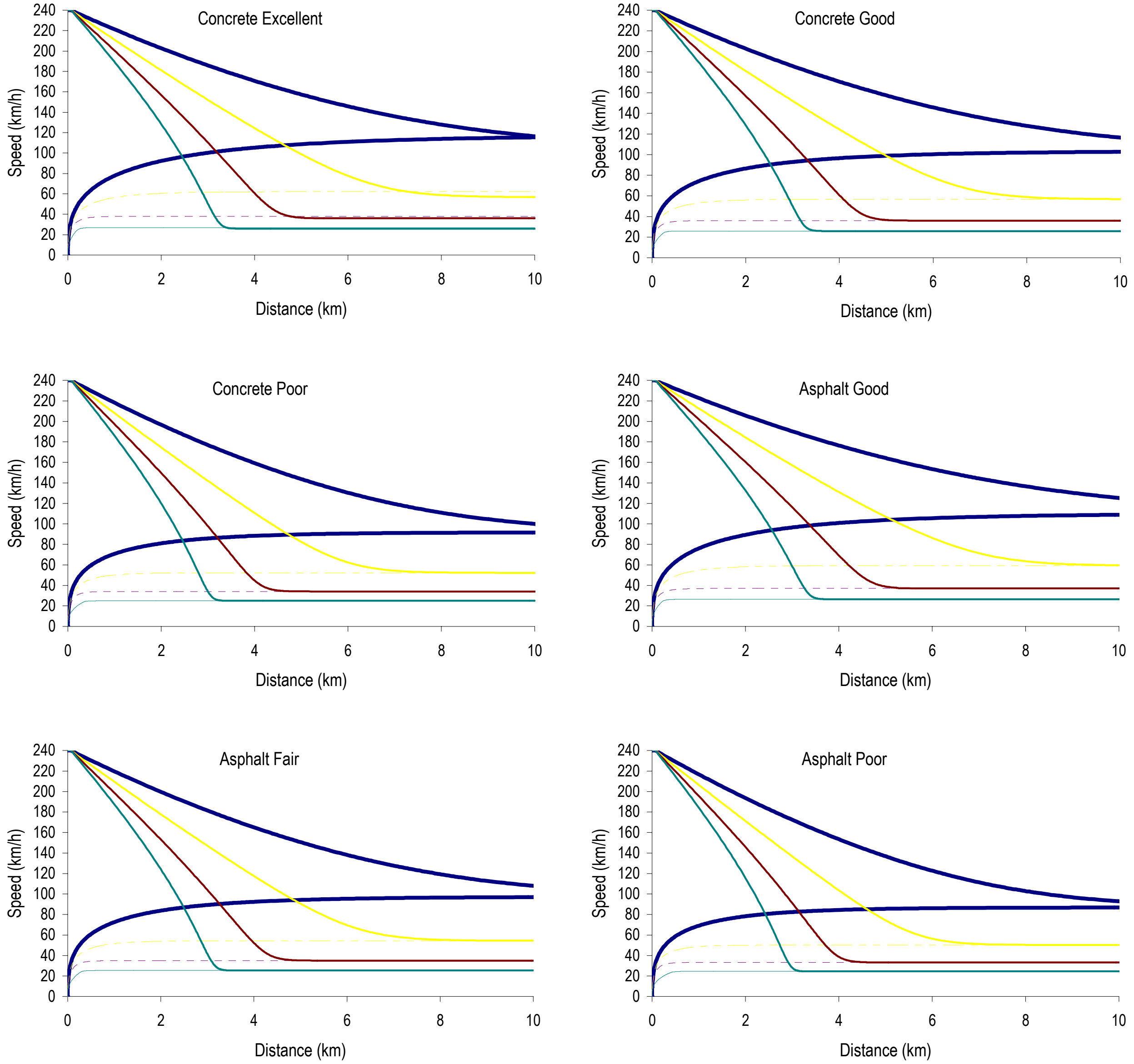


**Figure 10: Truck Performance Curves (Grade 0%, 2%, 4%, and 6%, Weight-to-Power Ratio 180 kg/kW, Power 335.7 kW)**

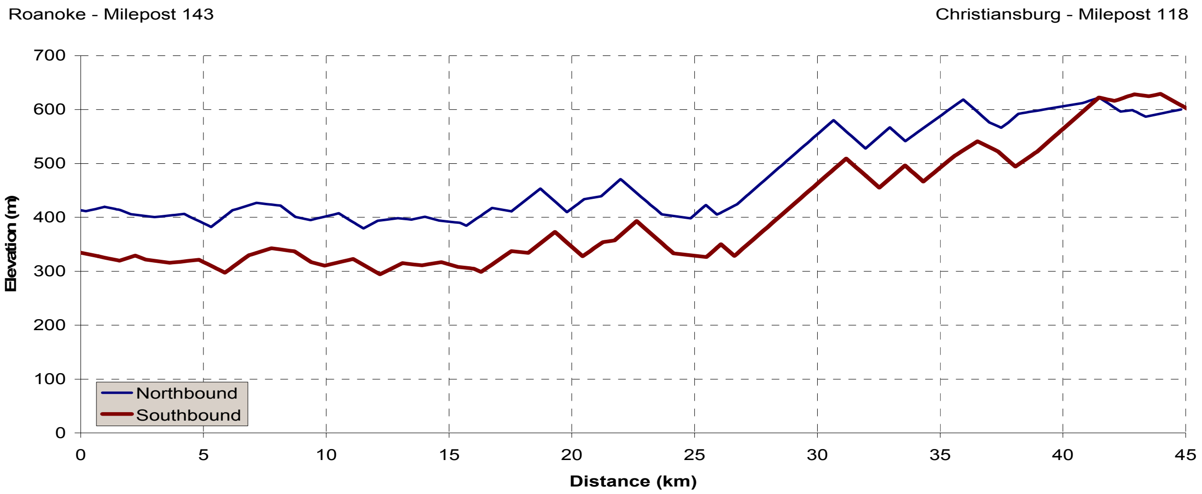


**Figure 11: I-81 Test Section Vertical Profile**

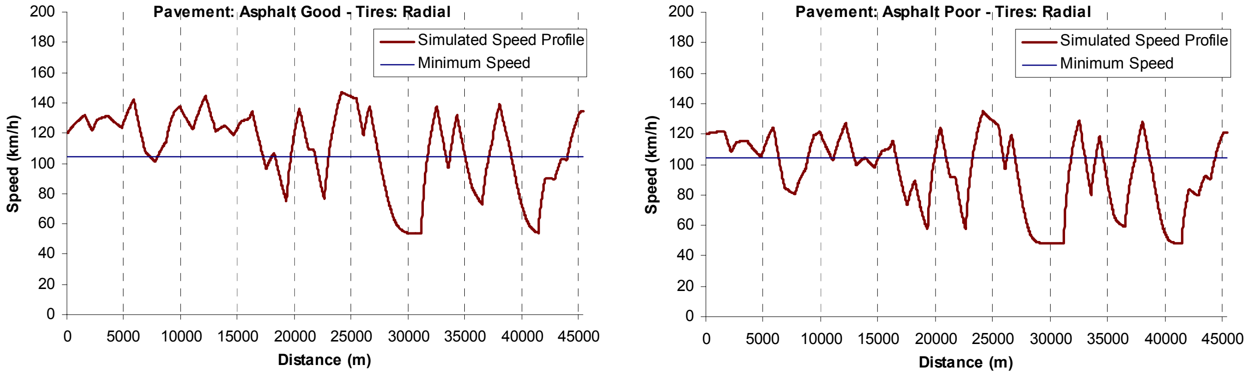


**Figure 12: Effect of Asphalt Condition on Truck Profile (I-81 Southbound – Truck 120 kg/kW)**

## APPENDIX A: DERIVATION OF EQUILIBRIUM SPEED

Given that the vehicle's acceleration is zero when it attains it's equilibrium speed ($v_m$), we can write the following mathematical equation:

$$a = \frac{F-R}{M} = 0 \qquad => \quad F = R$$
$$\therefore \quad \min(F_T, F_{\max}) = K_a v_m^2 + K_{r_1} v_m + K_{r_2} + R_g \qquad [18]$$

In most cases the introduction of the $\beta$ coefficient results in an $F_T$ that is less than $F_{max,}$. Considerng that $F_{\max} \geq \frac{K_T \beta}{v_0}$, Equation 18 can be simplified to Equation 19.

$$\frac{K_T \beta}{v_m} = K_a v_m^2 + K_{r1} v_m + K_{r2} + R_g \qquad [19]$$

According to Equation 6, the parameter $\beta$ can take two forms, depending on the vehicle's speed. Specifically, when the equilibrium speed ($v_m$) is larger than $v_0$, $\beta$ takes a value of 1.0; alternatively when $v_m$ is less than $v_0$, $\beta$ is equivalent to $\frac{1}{v_0}(1 + v - \frac{v}{v_0})$. Consequently, Equation 19 can be transformed to two equations, namely Equation 20 and 21, as follows:

$$\text{for}\, v_m \geq v_0\,,\; K_a v_m^3 + K_{r_1} v_m^2 + (K_{r_2} + R_g) v_m - K_T = 0 \qquad [20]$$

$$\text{for}\, v_m < v_0\,,\; K_a v_m^3 + K_{r_1} v_m^2 + (K_{r_2} + R_g - \frac{K_T}{v_0} + \frac{K_T}{v_0^2}) v_m - \frac{K_T}{v_0} = 0 \qquad [21]$$

From the mathematical view point, Equations 20 and 21 are third order polynomial equations in a single variable (the equilibrium speed $v_m$). Consequently, 3 roots to the equation can be computed of which 1 of these roots is real and the other 2 roots are complex. In some special cases, however, all three roots can be real. However, in order for all roots to be real the factor $K_a$ must be negative, which is impossible. Consequently, the equation form of Equations 20 and 21 offer a single real root that is computed using Equation 22 where the values of the constants $K_a$, $K_{r1}$, $b$, $c$, and $d$ depend on whether Equation 20 or 21 is utilized.

$$v_m = -\frac{K_{r1}}{3K_a} + \frac{\sqrt[3]{2}(-K_{r1}{}^2 + 3K_a c)}{3K_a b} - \frac{b}{3\sqrt[3]{2} \cdot K_a} \qquad [22]$$

Where:

$$b = \sqrt[3]{-27K_a^2 d - 2K_{r1}^3 - 9K_a K_{r1} c + \sqrt[2]{4(-K_{r1}^2 + 3K_a c)^3 + (-27K_a^2 d - 2K_{r1}^3 - 9K_a K_{r1} c)^2}}$$

$$c = \begin{cases} K_{r_2} + R_g & v_m \geq v_0 \\ K_{r_2} + R_g - \dfrac{K_T}{v_0} + \dfrac{K_T}{v_0^2} & v_m < v_0 \end{cases}$$

$$d = \begin{cases} -K_T & v_m \geq v_0 \\ -\dfrac{K_T}{v_0} & v_m < v_0 \end{cases}$$

In the case that $F_{max}$ is less than $F_T$ (i.e. $F_{\max} < \frac{K_T \beta}{v_0}$) then the equilibrium speed can be solved for using Equation 23 depending on whether the equilibrium speed exceeds the value $K_T\beta/F_{\max}$. Given that the equilibrium speed should be positive we only consider the positive root of the equation.

$$v_m = \begin{cases} \dfrac{-K_{r1} + \sqrt{K_{r1}^2 - 4K_a\left(K_{r2} + R_g - F_{\max}\right)}}{2K_a} & \forall\, v_m \leq \dfrac{K_T \beta}{F_{\max}} \\ -\dfrac{K_{r1}}{3K_a} + \dfrac{\sqrt[3]{2}(-K_{r1}{}^2 + 3K_a f)}{3K_a e} - \dfrac{e}{3\sqrt[3]{2} \cdot K_a} & \forall\, v_m > \dfrac{K_T \beta}{F_{\max}} \end{cases} \qquad [23]$$

Where:

$$e = \sqrt[3]{27K_a^2 K_T - 2K_{r1}^3 - 9K_a K_{r1} f + \sqrt[2]{4(-K_{r1}^2 + 3K_a f)^3 + (27K_a^2 K_T - 2K_{r1}^3 - 9K_a K_{r1} f)^2}}$$

$$f = K_{r_2} + R_g$$